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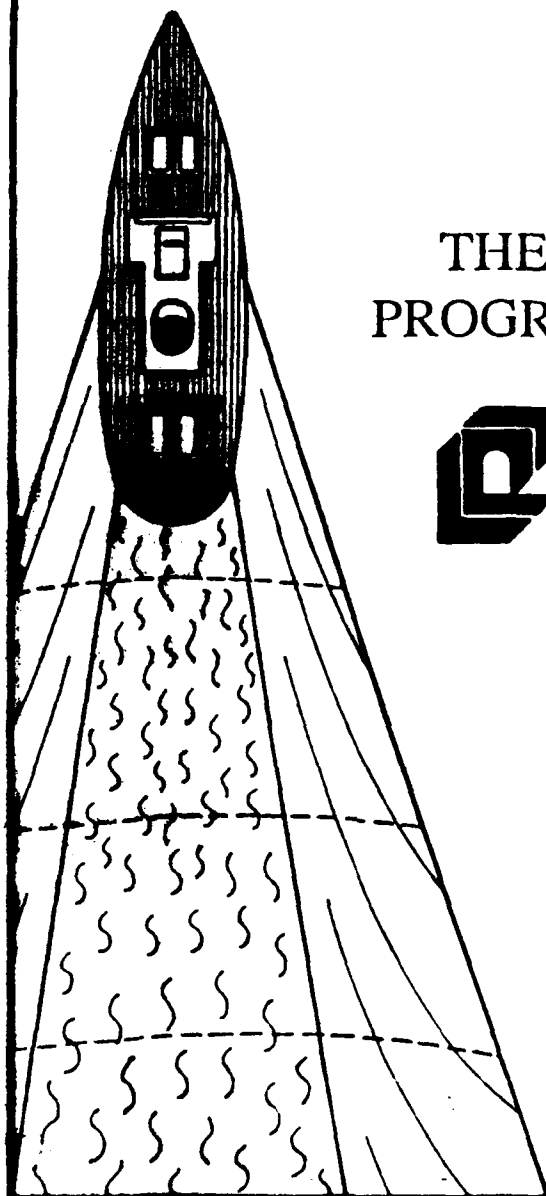
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WAKE PATTERNS COMPUTED BY A VORTEX METHOD

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Abstract

A wake is modelled by a vortex sheet carrying positive and negative circulation. The sheet's evolution is computed by the vortex-blob method. Initial conditions and circulation density for the vortex sheet are chosen to simulate some of the wake patterns observed in the soap-film experiments of Couder et.al.^{2,3}

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1. Introduction

At moderate Reynolds number, the wake behind a bluff body in a streaming flow forms a regular Kármán vortex street, consisting of two staggered rows of oppositely-signed vortices¹. These regular vortex streets have been observed in "natural" experimental situations, where no explicit forcing has been introduced. Experimentalists have also examined "forced" wakes, in which the bluff body is subjected to forced oscillations. In particular, Y. Couder and his associates^{2,3} have investigated the wake behind an oscillating solid cylinder in a thin soap-film. They have shown that various different vortex street patterns can form in the cylinder's wake when the forcing frequency is varied. The forced wakes observed by Couder et. al.^{2,3} differ from the classical Kármán vortex street in that they are dominated by "vortex-couples", i.e. pairs of counter-rotating vortices which propagate away from the wake's centerline.

Even though these experiments were effectively two-dimensional, a computational study of the problem would be very ambitious if it were to include the unsteady separation process as well as the wake's downstream development. The present paper has a more modest aim. Recently developed ideas for computing desingularized vortex sheet evolution⁴ will be applied to a spatially periodic model of the experiment. The effect of the unsteady experimental forcing will be simulated in the computation by choosing specific initial conditions for the vortex sheet. The circulation density will be chosen to simulate a free sinusoidal wake and the initial vortex sheet shape will be chosen so as to produce vortex-couples. The aim is to demonstrate that a desingularized vortex sheet model can simulate experimentally observed wake patterns. The model may therefore be a useful tool to complement analysis by more detailed fluid models, such as the Navier-Stokes equations.

Various types of vortex models and computational vortex methods have been used to study wakes. For example, the method of contour dynamics has been used to study a model in which the wake is represented by two layers of constant, oppositely-signed vorticity⁵. Point-vortex^{6,7}, vortex-blob^{8,9,10} and vortex-in-cell methods¹¹ have been used to study the evolution of two vortex sheets. Modelling a wake by two vortex layers or two vortex sheets is quite natural because in general, a bluff body has two separation points, each of which contributes a shear layer to the wake. However, the experiments of Couder et. al.^{2,3}, and others, suggest that in some instances, a wake can be modelled by a single vortex sheet carrying positive and negative circulation. This, for example, might be the case when the two separation points on the body are close to one another. Each complete

shedding cycle could be split into two parts, circulation predominantly of one sign being shed during each part. In effect, a single shear layer with circulation density of alternating sign could be shed into the wake. The present paper deals with a single vortex sheet model for a wake. Couder and Basdevant² have performed a numerical study of vortex-couples using a spectral method, and they give references to other numerical work as well.

The desingularized vortex sheet model and the numerical method are reviewed in section 2. Computed results are presented and compared with the experimental wake patterns of Couder et. al.^{2,3} in section 3. The results are discussed in section 4.

2. Numerical Method

The vortex structure forming the wake is represented by a vortex sheet $(x(a, t), y(a, t))$, which is periodic in the Lagrangian variable a . The circulation density along the curve is defined by a function $\sigma(a)$. The specific choice of $\sigma(a)$ depends upon the type of flow being modelled. In the present computations, $\sigma(a)$ will be chosen to have mean zero over one period, thereby modelling a wake in which equal amounts of positive and negative circulation are shed in each cycle.

Let $G(x, y)$ denote the Green's function for the Laplace equation in two dimensions. The stream function for the flow is expressed as,

$$\psi(x, y, t) = \int G(x - x(a, t), y - y(a, t))\sigma(a)da. \quad (1)$$

Velocity components are defined by $u = \psi_y, v = -\psi_x$, the partial derivatives being carried out by analytically differentiating under the integral sign in Eq. (1). The velocity of the vortex sheet is defined by $x_t = u, y_t = v$; the integrals here are evaluated on the curve and are interpreted as Cauchy principal value integrals. The problem specification is completed by choosing a circulation density $\sigma(a)$ and an initial shape $(x(a, 0), y(a, 0))$. This is simply a formulation of the equations governing vortex sheet evolution¹².

The calculations presented below use a desingularized periodic Green's function,

$$G(x, y; \delta) = \frac{-1}{4\pi} \log(\cosh 2\pi y - \cos 2\pi x + \delta^2), \quad (2)$$

in which δ is an artificial smoothing parameter⁴. Positive values $\delta > 0$ diminish the vortex sheet's short wavelength instability and allow the curve to roll up smoothly. In the computations presented below the value $\delta = 0.3$ was used. The curve is discretized by a finite number of points per wavelength, corresponding to a uniform mesh in the parameter a . The integrals that define the curve's velocity are approximated by the trapezoid rule. The

ordinary differential equations for the point positions are integrated forward in time by the fourth order Runge-Kutta method. Points are inserted adaptively as the curve stretches, in order to maintain spatial resolution¹³. This is the vortex-blob or desingularization method for computing vortex sheet roll-up^{4,8,9}.

3. Results

Figures 1-3 show the computed results and compare them to the experimental wake patterns of Couder et. al.^{2,3}. The experimental wakes were produced in a thin soap-film by moving a solid cylinder in the plane of the film. The system was forced by oscillating the solid cylinder at a given frequency. The flow visualization was performed by detecting changes in the soap-film thickness. In the experiments reproduced below, the downstream direction is to the left. For the computations, the initial shape $(x(a,0), y(a,0))$ and circulation density $\sigma(a)$ were taken to be simple patched combinations of a fundamental sinusoid and a subharmonic. In each case, these functions were chosen so as to simulate the experimental wake patterns (the specific choices are given in the Figure captions). The calculations were performed over one spatial period, but four periods are plotted in the Figures.

The experimental wake shown at the bottom of Figure 1 is "natural" in the sense that no forcing was explicitly supplied. In this case a regular Kármán vortex street formed, consisting of vortices that alternate in sign. The vortices are staggered a small distance on either side of the wake. The width of the wake grows slowly as a function of distance downstream from the body. The computational circulation density was taken to be $\sigma(a) = \sin 2\pi a$, in order to produce equal strength vortices that alternate in sign. The staggered position of the vortices was obtained by putting a transverse sinusoidal perturbation, in phase with the circulation density, into the curve's initial shape. The computation develops in time into a regular vortex street which resembles the experiment.

Couder et. al.^{2,3} found that different patterns formed when the cylinder was forced at the natural shedding frequency (Figure 2) and at a slightly different frequency (Figure 3). The forced experimental wakes are dominated by vortex-couples, each consisting of a pair of counter-rotating vortices. In Figure 2, the couples travel away from the wake centerline on one side, in an oblique upstream direction. The width of the wake increases more rapidly than in Figure 1. To obtain the vortex-couples, the computation in Figure 2 used the same circulation density as before, but the initial shape was slightly more involved, containing both a transverse and a longitudinal perturbation.

In Figure 3, the vortex-couples travel away from the centerline on both sides. The width of the wake increases more rapidly than in the previous cases. The computational circulation density and initial shape were constructed by patching together two of the functions used to obtain Figure 2. Again, the computations qualitatively resemble the experiment. It may be noted in Figure 3a that the curve's initial shape has a slope discontinuity at $a = 0$ and at $a = 0.5$. This singularity however appears to be smoothed out at later times.

4. Discussion

The desingularized vortex sheet computations shown here used the value $\delta = 0.3$ for the smoothing parameter. Previous numerical work on the effect of varying δ shows that the curve's large-scale properties, such as the spiral position and size, are only weakly dependent upon the precise value of δ ^{4,9,13}. The curve rolls up more quickly and more tightly as $\delta \rightarrow 0$, apparently converging to a spiral with an infinite number of turns in the limit. Similar behaviour has been demonstrated for the vortex-in-cell method applied to vortex sheet roll-up, where the smoothing is provided by computing the particles' velocity on an underlying regular grid¹⁴. Rigorous results concerning the limit $\delta \rightarrow 0$ for vortex sheets have been obtained^{15,16}, although complete justification is not currently available.

None of the computations shown here required more than 4 minutes of cpu time on a CRAY X-MP/48 computer. Couder et. al.^{2,3} have observed more disordered wake patterns at other forcing frequencies than those reproduced in Figures 1-3. Such patterns would be more expensive to simulate by the present method.

A vortex-dipole sheet model for a wake has recently been proposed¹⁷. In that work, computational vortex-dipoles were used to represent oppositely-signed wake vorticity that originates in the boundary layers upstream from a separation point. This effect has not been included here. The positive and negative circulation dealt with in the present work is associated with two distinct separation points on a bluff body.

It should be noted that the initial shape and circulation density in each computation was chosen to give qualitative agreement with the experimental flow visualization. A further requirement was that the choices be as simple as possible. The actual functions used were determined by trial and error. The aim was to demonstrate that the desingularized vortex sheet model has the capability of simulating wake patterns observed in experiment. The model obviously neglects certain physical mechanisms in the experiment. For example, these periodic computations do not describe the unsteady separation process and the spatial growth of the wake. Questions then arise : How can such a crude model yield good

qualitative agreement with experiment? Can the model be extended to include some of the neglected physical mechanisms? If so, can one obtain quantitative agreement between the model and experiment? These questions cannot be answered at present, but they are worth pursuing. Along these lines, a desingularized model for vortex sheet separation at a sharp edge is being developed.

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Figure Captions

FIG. 1

- a) Initial shape $x(a, 0) = a, y(a, 0) = 0.2 \sin 2\pi a$, circulation density $\sigma(a) = \sin 2\pi a$, smoothing parameter $\delta = 0.3$.
- b) Experiment reproduced from Couder and Basdevant². With no forcing explicitly imposed, a regular Kármán vortex street formed.

FIG. 2

- a) Initial shape $x(a, 0) = a + 0.1 \sin 2\pi a, y(a, 0) = 0.1 \sin 2\pi a$, circulation density $\sigma(a) = \sin 2\pi a$, smoothing parameter $\delta = 0.3$.
- b) Experiment reproduced from Couder and Basdevant². Vortex couples propagate obliquely away from the centerline on one side. This pattern resulted from forcing imposed at the natural shedding frequency.

FIG. 3

- a) Initial shape $x(a, 0) = a + 0.05 \sin 2\pi a, y(a, 0) = \begin{cases} a + 0.05 \sin 4\pi a, & 0 \leq a \leq 0.5 \\ a - 0.05 \sin 4\pi a, & 0.5 \leq a \leq 1 \end{cases}$
circulation density $\sigma(a) = \begin{cases} \sin 4\pi a, & 0 \leq a \leq 0.5 \\ -\sin 4\pi a, & 0.5 \leq a \leq 1 \end{cases}$, smoothing parameter $\delta = 0.3$.
- b) Experiment reproduced from Couder and Basdevant². Vortex couples propagate obliquely away from the centerline on both sides. This pattern resulted from forcing imposed at slightly other than the natural shedding frequency.

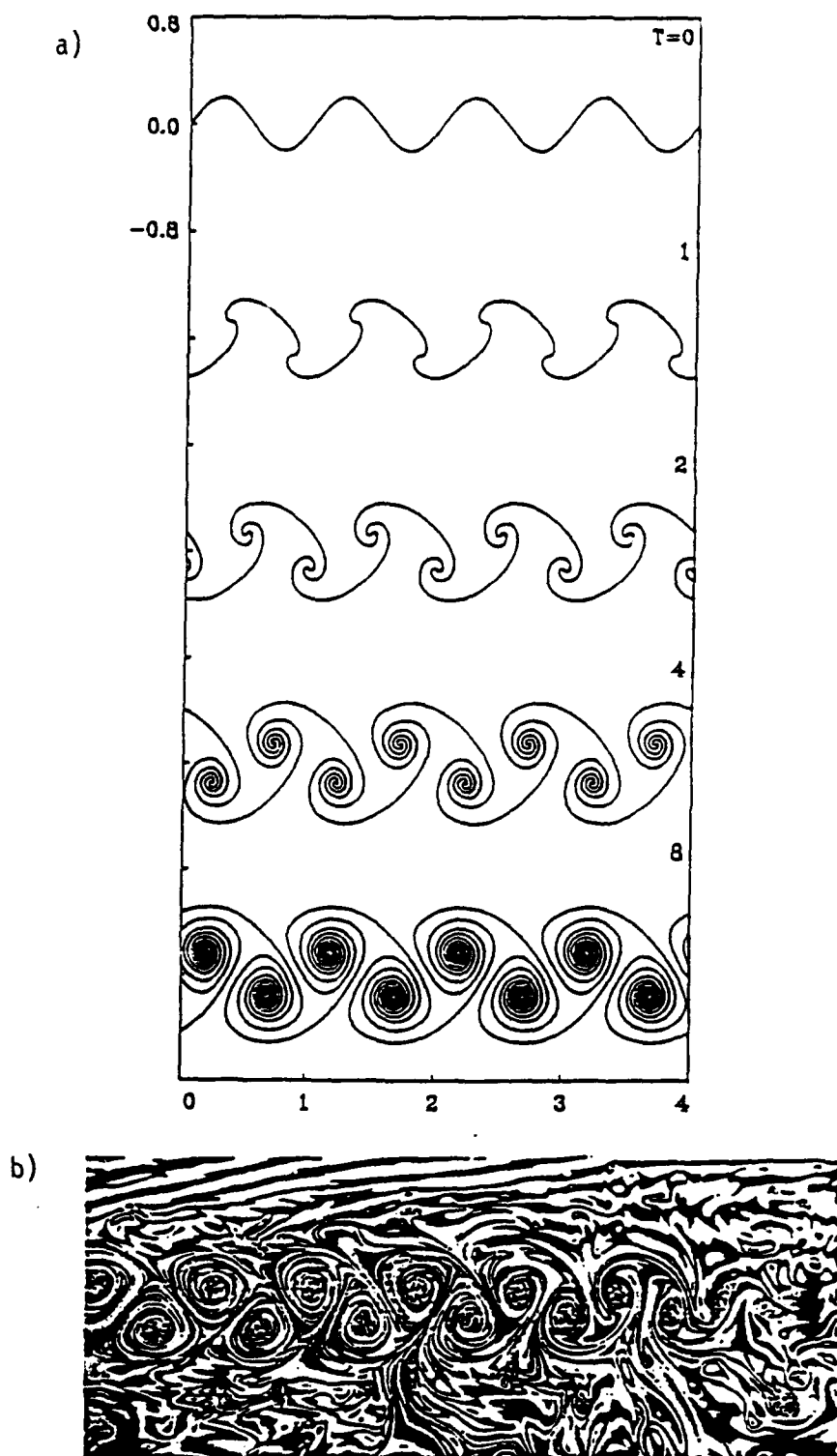
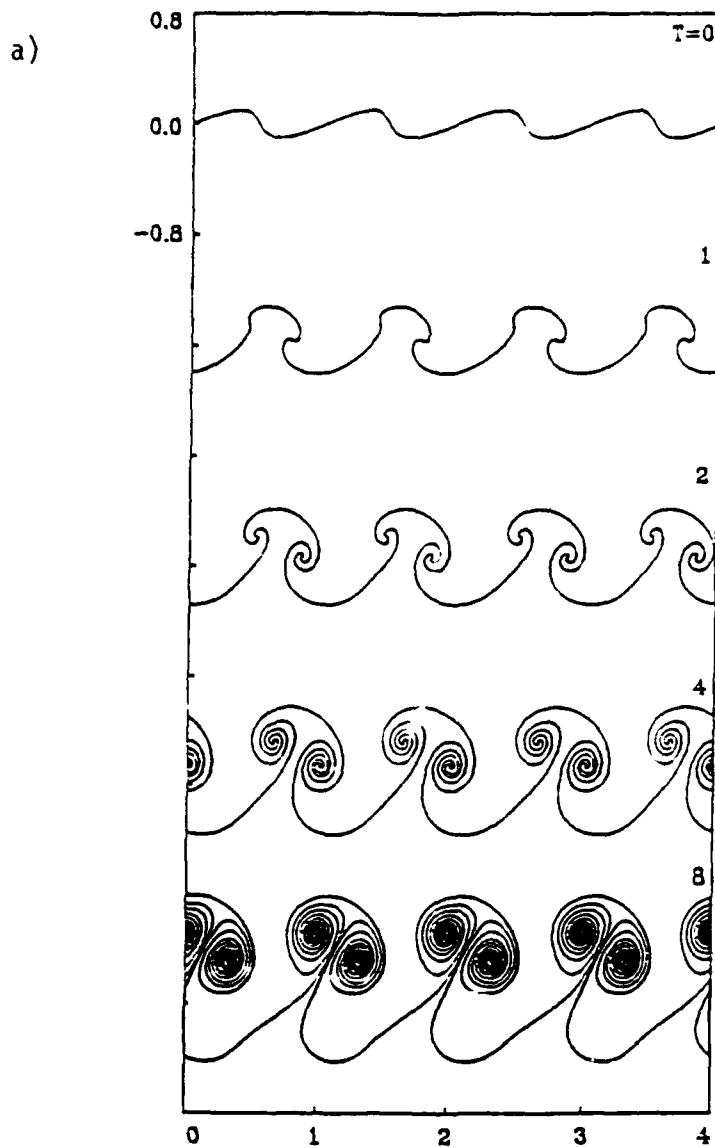


FIG. 1

a) Initial shape $x(a, 0) = a, y(a, 0) = 0.2 \sin 2\pi a$, circulation density $\sigma(a) = \sin 2\pi a$, smoothing parameter $\delta = 0.3$.

b) Experiment reproduced from Couder and Basdevant². With no forcing explicitly imposed, a regular Kármán vortex street formed.



b)



FIG. 2

a) Initial shape $x(a, 0) = a + 0.1 \sin 2\pi a$, $y(a, 0) = 0.1 \sin 2\pi a$, circulation density $\sigma(a) = \sin 2\pi a$, smoothing parameter $\delta = 0.3$.

b) Experiment reproduced from Couder and Basdevant². Vortex couples propagate obliquely away from the centerline on one side. This pattern resulted from forcing imposed at the natural shedding frequency.

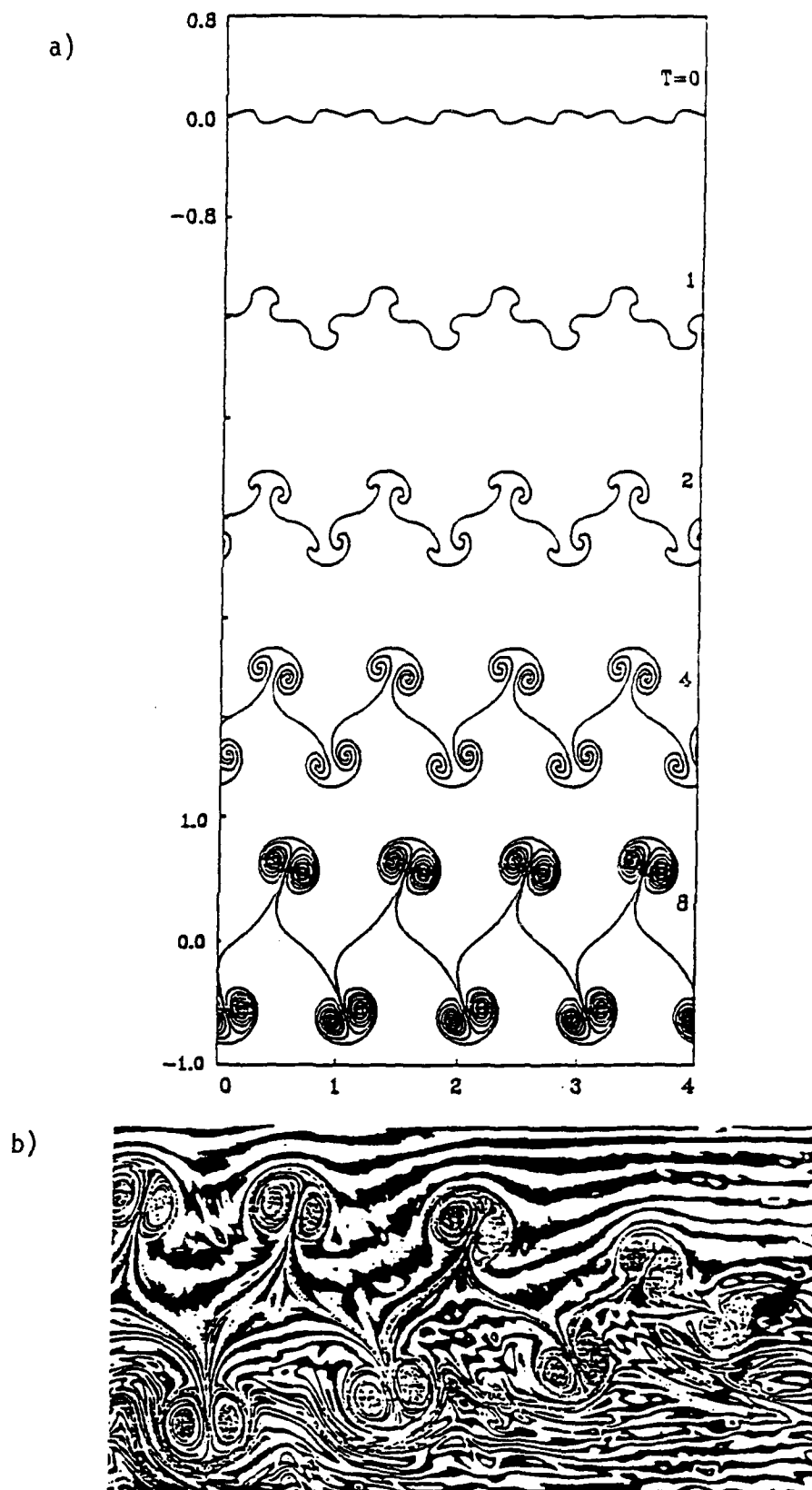


FIG. 3

a) Initial shape $x(a, 0) = a + 0.05 \sin 2\pi a$, $y(a, 0) = \begin{cases} a + 0.05 \sin 4\pi a, & 0 \leq a \leq 0.5 \\ a - 0.05 \sin 4\pi a, & 0.5 \leq a \leq 1 \end{cases}$
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b) Experiment reproduced from Couder and Basdevant². Vortex couples propagate obliquely away from the centerline on both sides. This pattern resulted from forcing imposed at slightly other than the natural shedding frequency.